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Possible Implications of Asymmetric Fermionic Dark Matter for Neutron Stars

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Abstract

We consider the implications of fermionic asymmetric dark matter (ADM) for a “mixed neutron star” composed of ordinary baryons and dark fermions. We find examples, where for a certain range of dark fermion mass – when it is less than that of ordinary baryons – such systems can reach higher masses than the maximal values allowed for ordinary (“pure”) neutron stars. This is shown both within a simplified, heuristic Newtonian analytic framework with non-interacting particles and via a general relativistic numerical calculation, under certain assumptions for the dark matter equation of state. Our work applies to various dark fermion models such as mirror matter models and to other models where the dark fermions have self interactions.

I. INTRODUCTION

Cold dark matter (CDM), favored by most astrophysical and cosmological observations, can be realized in symmetric or asymmetric scenarios. In the first class of models, dark matter is made of stable X particles and an equal amount of stable \bar{X} antiparticles of mass m_X . In the early universe, these were in thermal equilibrium and their residual abundance Ω_X is fixed, at the “freeze-out” value, when the rate of the Hubble expansion overcomes that of $\bar{X} - X$ annihilation. A prototypical example, which has been extensively studied, is provided by supersymmetric models with R-parity conservation, where the lightest superpartner is stable and plays the role of the dark matter of the universe.

As yet no light sub-TeV SUSY partners have been discovered at the LHC, and searches for electrons, positrons or photons from annihilations in clumps of DM in and around our Galaxy, do not provide solid “indirect” evidence for symmetric massive DM. Moreover, the ongoing direct underground searches put very strong bounds on the scattering cross sections of massive X’s on nuclei. In the symmetric case, accretion of DM particles onto the sun accelerates the rate of particle-antiparticle annihilation. The resulting photons, electrons, etc., are all trapped in the star. However, for massive DM particles, looking in ICE-CUBE (the large km³ Cerenkov radiation detector near the south-pole) for the resulting UHE neutrinos is an excellent indirect detection method. The fact that no such energetic neutrinos have been detected constrains symmetric DM models. Consequently there has been, in recent years, a renewed interest in a second class of models: the asymmetric dark matter (ADM) models. In such models, the relic ADM density is determined in a manner analogous to that of ordinary baryonic matter, not by the freeze-out of DM annihilation, as in the symmetric case. An excess of dark fermions (over the antifermions) remains after the annihilation of most antiparticles. The required dark matter density in such models is readily achieved if the ratio of the ADM particles mass and that of ordinary baryons is tuned inversely with the corresponding ratio of asymmetries. Many examples of such models have been proposed over the years [1]. Here we consider variants in which the dark matter particle is rather light with mass in the sub- GeV range. Scattering of such light CDM on most detector materials yields

recoil energies ~ 0.1 KeV which are below the existing experimental thresholds. Hence the present upper bounds on the X-N scattering cross-sections do not apply. Also the stringent indirect upper bounds from missing energy searches at the collider [2] apply for massive mediators of the X-nucleon interactions - and do not apply if the exchange of a relatively light "dark photon" mediates X-N scattering, as is the case in several asymmetric dark matter models. This may allow σ_{X-N} of order 10^{-34} cm^2 - which is high enough to be relevant in astrophysical settings and yet is 10 orders of magnitude smaller than the intra- species cross-sections of ordinary matter and potentially of dark matter.

For our purpose, in this paper, it is useful to consider a class of models for ADM proposed in [3] and its possible variants. These contain an additional sector mirroring our universe. The mirror sector consists of particles and forces related to those of the familiar standard model by a mirror symmetry. As a result, there are no new parameters in the model prior to gauge symmetry breaking [4]. In generic mirror models, an important constraint comes from BBN due to the presence of three extra neutrinos and an extra photon. One way to avoid this constraint is to assume that the temperature of the mirror sector is smaller than that of the familiar sector [5]. An alternative possibility detailed in [3] is to have the symmetry breaking in the mirror sector sufficiently different from that in the familiar sector so that all the mirror neutrinos and mirror photon are heavy and have decayed by the BBN epoch and only the mirror neutrons survive constituting the dark matter. Our considerations are independent of the model details of [3] and could be applied to variants of the model where the mirror photon is very light, e.g. less than an eV. The details of large scale structure formation depend on the specific model for the asymmetric dark matter. Being self interacting the dark matter will no longer provide collision-less dark halos with many possible cosmological ramifications [6].

In asymmetric DM models, the dark matter particles can accumulate in astrophysical objects and alter their properties. The goal of the present paper is to study the effect of such accumulation on neutron star properties. Similar studies for the case of scalar ADM have been reported in several papers [7], where Bose condensation plays an important role. The situation for the case of fermionic dark matter is however very different due to the Pauli exclusion principle and our goal is to make some remarks on this

case. We find that under certain conditions, the mass of the mixed neutron- DM star can exceed the Chandrasekhar-like mass limit for ordinary neutron stars. The recent discovery of a $2M_{\odot}$ binary radio pulsar [8], already severely constrains nuclear matter equations of state, if it is a “pure” neutron star and possible future observation of such neutron stars with higher masses would be very difficult to reconcile with standard hadronic physics but, as we show in this paper, such higher mass neutron stars seem to be more easily realized as mixed neutron stars.

Another result of our discussion is that, for a mixed neutron star with two species which interact with each other only via gravitational interactions, requiring stability (see sec. 4) imposes an interesting scaling relation between the number and energy density and pressure. Such a relation will constrain the density profiles of the model as well as the number distribution of the two species.

This paper is organized as follows: in sec. II, we discuss a Newtonian model for a mixed neutron star; in sec. III, we present the general relativistic treatment of the mixed neutron star containing both ordinary and dark fermions. In sec. IV, we discuss the implications of stability (extremum with respect to variations of mass-energy density keeping the total number of particles fixed) for the mixed neutron star; see Eq. (38). We find the interesting relation cited above among pressure, density and the particle number density in the two sectors. In sec. V, we present an illustrative example where we employ the same equation of state for the familiar sector and the dark sector to discuss the impact on neutron star mass. In sec. VI, we present some astrophysical discussion and we conclude in sec. VII.

II. MAXIMAL MASS OF MIXED NEUTRON STARS: HEURISTIC DISCUSSION

Before proceeding to a detailed analysis, let us start with a heuristic discussion based on Newtonian intuition for a mixed neutron star ignoring nuclear physics effects. We generalize to the present mixed case, the discussion in [9] which estimated the maximal mass of an ordinary neutron star. The total energy of a mixed neutron star is the sum of

the relativistic Fermi energy and the gravitational energy: $E = E_F + E_G$ where:

$$E_F = \beta \frac{\hbar c}{R_1} N_1^{4/3} + \beta \frac{\hbar c}{R_2} N_2^{4/3}, \quad E_G = -\frac{1}{8\pi G} \int_0^\infty \left(\frac{Gm(r)}{r^2} \right)^2 4\pi r^2 dr \quad (1)$$

Here, β is a coefficient of order unity, N_i ($i = 1, 2$) denote the total number of baryons ($i = 1$) and dark fermions ($i = 2$) and $m(r)$ is the mass enclosed within a sphere of radius r . We use a Newtonian approximation for $m(r)$ and assume further (as justified a posteriori by the detailed numerical calculations) that the energy and mass densities of the two species can be approximated as constants, in their respective spheres of radii R_1, R_2 . Thus, we have

$$m(r) = M_1 \left(\frac{r}{R_1} \right)^3 + M_2 \left(\frac{r}{R_2} \right)^3, \quad r \leq R_1 \quad (2)$$

$$m(r) = M_1 + M_2 \left(\frac{r}{R_2} \right)^3, \quad R_1 \leq r \leq R_2 \quad (3)$$

$$m(r) = M_1 + M_2, \quad r \geq R_2 \quad (4)$$

Substituting the above expression for $m(r)$ in the integral expressing E_G and performing the integration over the inner, the intermediate and the outer regions we find:

$$E_G = -\frac{3}{5} \frac{GM_1^2}{R_1} - \frac{3}{5} \frac{GM_2^2}{R_2} - \frac{3}{2} \frac{GM_1 M_2}{R_2} + \frac{3}{10} \frac{GM_1 M_2}{R_2} \left(\frac{R_1}{R_2} \right)^2 \quad (5)$$

For the case where R_2 exceeds R_1 , the last term is small compared to the previous term and can be neglected. Hence, we get

$$E = \beta m_1^{-4/3} \frac{\hbar c}{R_1} M_1^{4/3} - \frac{3}{5} \frac{GM_1^2}{R_1} + \beta m_2^{-4/3} \frac{\hbar c}{R_2} M_2^{4/3} - \frac{3}{5} \frac{G(M_2^2 + 2.5M_1 M_2)}{R_2} \quad (6)$$

where in the spirit of the Newtonian approximation we used $M_i = N_i m_i$, $i = 1, 2$, where $m_{1,2}$ are the masses of the familiar neutron and the dark fermion respectively.

Following [9], one can argue that the sums of the coefficients multiplying $1/R_1$ and $1/R_2$ should be positive in order to avoid gravitational collapse to a black hole. Thus one gets

$$M_1 - M_0 \leq 0 \quad (7)$$

and

$$M_2^2 - M_2^{4/3} M_0^{2/3} \left(\frac{m_1}{m_2} \right)^{4/3} + 2.5 M_1 M_2 \leq 0 \quad (8)$$

with

$$M_0 = \left(\frac{5\beta}{3}\right)^{3/2} \frac{m_{pl}^3}{m_1^2} \quad (9)$$

denoting the maximal mass of a pure neutron star. Its value is of the order of a solar mass. Some realistic nuclear equations of state for familiar neutrons, including the one employed in the next sections, allow values of $M_0 \approx 2.5M_\odot$.

The total mass of the mixed neutron star is

$$M = M_1 + M_2 \quad (10)$$

The above constraints correspond to the case where the neutrons-sphere is within the outer radius, R_2 . We can consider the opposite case in which the dark matter sphere is enclosed within the neutron-sphere. In this case we will get

$$M_2 - M_0 \left(\frac{m_1}{m_2}\right)^2 \leq 0 \quad (11)$$

$$M_1^2 - M_1^{4/3} M_0^{2/3} + 2.5M_1 M_2 \leq 0 \quad (12)$$

where M_0 is the same as defined above.

Note that we do not obtain constraints on the radii, because the relativistic limit for the Fermi energies was adopted. Had we taken the general expression for the Fermi energies and minimized with respect to each radius, we would have obtained also constraints on the radii. However, this would have increased the complexity of the heuristic analytic estimates. Furthermore we find a similar result when solving numerically the general relativistic (GR) mixed star model. It follows from the above constraints that the maximal mass of an ordinary neutron star is M_0 and that of a pure dark (mirror) analog of a neutron star is $M_0(m_1/m_2)^2$. Therefore, this mass will be larger than that of an familiar neutron star only if the mass of the dark (mirror) baryon is smaller than the mass of the neutron. Since having degenerate neutron and dark matter fermion mass decreases the maximal mass of the neutron star, this holds *a fortiori* for mixed neutron stars. In what follows we use an illustrative value of $m_2 = \frac{1}{2}m_1$. The blue shaded area in Fig. 1 marks the region in the $M_1 - M_2$ plane allowed by equations (7, 8) for $m_2 = \frac{1}{2}m_1$. The yellow shaded area marks the region in the $M_1 - M_2$ plane allowed by equations (11, 12). The masses are expressed in units of M_0 .

The blue shaded area corresponds to the case where the dark matter sphere extends beyond that of the familiar neutron sphere. This region is particularly interesting, as it allows a total mass exceeding the maximal mass of a pure neutron star. Note that the maximally allowed dark fermion mass is always smaller than the neutron mass for total mass to exceed the Chandrasekhar limit, M_0 with the “ultimate” limit on the mixed neutron star mass being four times that of the pure neutron star for dark matter mass being half that of the familiar neutron. Clearly the ultimate limit depends on the ratio m_2/m_1 .

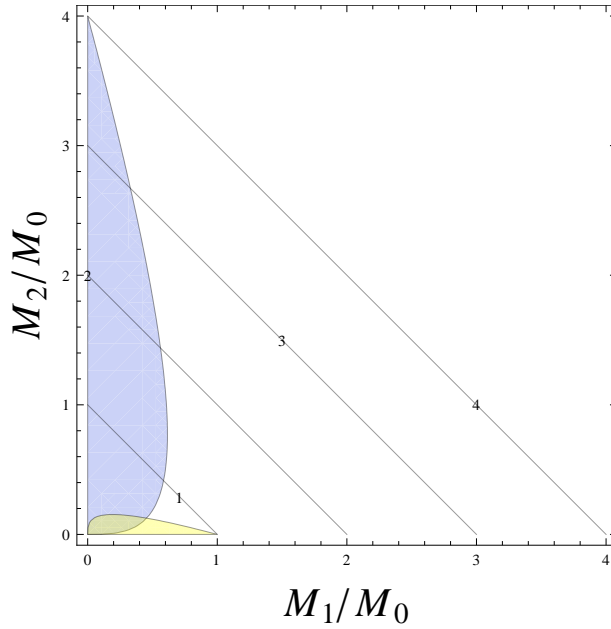


FIG. 1. Allowed regions in the $M_1 - M_2$ plane for $m_2 = \frac{1}{2}m_1$. *Blue shaded area:* $R_2 > R_1$. *Yellow shaded area:* $R_1 > R_2$. *Diagonal lines:* loci of total mass.

III. DYNAMICS OF A MIXED NEUTRON STAR

Encouraged by the above heuristic results, we proceed to a fully general relativistic (GR) discussion. The energy momentum tensor of a mixture of two non-interacting ideal fluids:

$$T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} = (\rho_1 + p_1)u_1^\mu u_1^\nu - p_1 g^{\mu\nu} + (\rho_2 + p_2)u_2^\mu u_2^\nu - p_2 g^{\mu\nu} \quad (13)$$

where $\rho_{1,2}$ and $p_{1,2}$ are the densities and pressures of the familiar and mirror matter. We consider here an ideal case in which the interaction between DM and familiar fermions is sufficiently weak that each tensor $T_{1,2}$ is separately conserved.

We look for a spherically symmetric static solution of the Einstein field equations for the two-fluid “mixed neutron star”. Since we address here a mixture of two fluids, we rederive the hydrostatic equilibrium equations for this case, starting with the Einstein field equations[10]. The line element squared of a spherically symmetric static metric can be written in the Schwarzschild coordinates (t, r, θ, ϕ) as

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = e^{2\phi(r)} c^2 dt^2 - e^{2\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \quad (14)$$

The $\alpha = \beta = t$ and the $\alpha = \beta = r$ equations, respectively, are

$$\frac{1}{r^2} \frac{d}{dr} (r(1 - e^{-2\lambda(r)})) = 8\pi \frac{G}{c^2} T_t^t = 8\pi \frac{G}{c^2} (\rho_1 + \rho_2) \quad (15)$$

$$-r^{-2} + e^{-2\lambda(r)} \left(r^{-2} + 2r^{-1} \frac{d\phi(r)}{dr} \right) = 8\pi \frac{G}{c^2} T_r^r = -8\pi \frac{G}{c^2} (p_1 + p_2) \quad (16)$$

We also have the separate two covariant conservation equations of the energy momentum tensors:

$$T_i^{\mu\nu}{}_{;\nu} = 0, \quad i = 1, 2 \quad (17)$$

The field equation (15) immediately yields

$$e^{-2\lambda(r)} = \left(1 - 2 \frac{G}{c^2} \frac{m(r)}{r} \right) \quad (18)$$

where $m(r)$ is the mass enclosed within r and is given by

$$m(r) = \int_0^r 4\pi \left((\rho_1(r') + \rho_2(r')) \right) r'^2 dr' \quad (19)$$

The two covariant conservation equations of the energy momentum tensors, equation (17) take the form:

$$\frac{d\phi(r)}{dr} = -\left(\rho_1(r) + p_1(r)\right)^{-1} \frac{dp_1(r)}{dr} \quad (20)$$

$$\frac{d\phi(r)}{dr} = -\left(\rho_2(r) + p_2(r)\right)^{-1} \frac{dp_2(r)}{dr} \quad (21)$$

Combining them with the field equation (16) leads to a hydrostatic equilibrium equation for each of the species:

$$\frac{dp_1(r)}{dr} = -G\left(\rho_1(r) + p_1(r)\right) \frac{m(r) + 4\pi r^3\left(p_1(r) + p_2(r)\right)}{r\left(r - 2Gc^{-2}m(r)\right)} \quad (22)$$

$$\frac{dp_2(r)}{dr} = -G\left(\rho_2(r) + p_2(r)\right) \frac{m(r) + 4\pi r^3\left(p_1(r) + p_2(r)\right)}{r\left(r - 2Gc^{-2}m(r)\right)} \quad (23)$$

These equations imply that each fluid satisfies its own hydrostatic equilibrium equation which is of the form of a modified TOV (Tolman, Oppenheimer, Volkoff) equation [11], [12]. In this paper, we assume that the two fluids are coupled only through gravity.

Given the two equations of state, and the two central energy densities, the TOV equations (22, 23) are integrated up to $r = R_1$ where $p_1(R_1) = 0$. Species 1 is confined within this radius. From this radius on, only TOV2 (equation 23) is integrated out to the radius R_2 where $p_2(R_2) = 0$, which is the outer radius of the complete mixed neutron star.

Once the solutions of equations (15-16) are obtained, equations (20-23) are solved with the boundary condition $\phi(R_2) = \frac{1}{2} \ln\left(1 - 2\frac{G}{c^2} \frac{M}{R_2}\right)$ with $M = m(R_2)$ being the mass of the mixed neutron star. This boundary condition is imposed by demanding that the inner solution be matched to the external Schwarzschild solution for which $\phi = -\lambda$.

In this way, a two-parameter (namely the two central densities) family of static models is obtained. In contrast, ordinary neutron star models form a one-parameter (one central density) family of solutions.

It is interesting that, combining the two covariant conservation laws of fermion number densities n_i

$$(n_i u_i^\alpha)_{;\alpha} = 0, \quad i = 1, 2 \quad (24)$$

(where u_i^α are the components of the relativistic velocity four-vector) with the two covariant conservation equations of energy momentum tensors (17) we find the isentropic relations for each of the fluids

$$\frac{d\rho_i}{\rho_i + p_i} = \frac{dn_i}{n_i}, \quad i = 1, 2 \quad (25)$$

Using Eq.(20, 21) [13], and noting that $p_i(R_i) = 0$, these equations lead to

$$n_1(r) = m_1^{-1} e^{-\phi(R_1) + \phi(r)} (\rho_1(r) + p_1(r)), \quad n_2(r) = m_2 e^{-\phi(R_2) + \phi(r)} (\rho_2(r) + p_2(r)) \quad (26)$$

In turn these constraints imply that inside the inner neutron radius R_1 , where two species coexist and we have the relation,

$$e^{+\phi(R_1)} \frac{n_1(r)m_1}{\rho_1(r) + p_1(r)} = e^{+\phi(R_2)} \frac{n_2(r)m_2}{\rho_2(r) + p_2(r)} \quad (27)$$

IV. STABILITY OF THE MIXED NEUTRON STAR

In what follows we show that the mass stability theorem [14] summarized in Weinberg's book [15] can be extended for a mixed neutron star (and in effect more generally for any mixed star constructed out of two non-interacting fluids). This imposes constraints on the equilibrium mass and density distributions of the dark matter relative to familiar neutrons in a neutron star.

Theorem

For quasistatic spherically symmetric structures with fixed given total baryon numbers (of the neutrons and the the dark mirror baryons), the total mass is stationary for variations of the the two energy densities $\rho_1(r)$, $\rho_2(r)$ if and only if the two equilibrium equations are satisfied.

As in the case of a single fluid, one considers the case where the entropy per baryon is uniform. Using the Lagrange multipliers method we explore the implications of demanding that

$$\delta M - \lambda_1 \delta N_1 - \lambda_2 \delta N_2 = 0 \quad (28)$$

where

$$M = \int_0^\infty 4\pi r^2 \left(\rho_1(r) + \rho_2(r) \right) dr$$

and

$$N_i = \int_0^\infty 4\pi r^2 n_i(r) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1/2} dr, \quad i = 1, 2$$

with $m(r)$ the mass enclosed within the radius r given by

$$m(r) = \int_0^r 4\pi r'^2 \left(\rho_1(r') + \rho_2(r') \right) dr'$$

Combining the above equations, one gets

$$\begin{aligned} & \int_0^\infty 4\pi r^2 \left(\delta\rho_1(r) + \delta\rho_2(r) \right) dr - \\ & \int_0^\infty 4\pi r^2 \left(\lambda_1 \delta n_1(r) + \lambda_2 \delta n_2(r) \right) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1/2} dr \\ & - G/c^2 \int_0^\infty 4\pi r \left(\lambda_1 n_1(r) + \lambda_2 n_2(r) \right) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-3/2} \delta m(r) dr = 0 \end{aligned} \quad (29)$$

The uniform entropy conditions, for each of the fluids, $\frac{d\rho_i}{\rho_i + p_i} = \frac{dn_i}{n_i}$, $i = 1, 2$

imply that

$$\frac{\delta n_i(r)}{n_i(r)} = \frac{\delta \rho_i(r)}{\rho_i(r) + p_i(r)}, \quad i = 1, 2 \quad (30)$$

Substituting $\delta m(r) = \int_0^r 4\pi r'^2 \left(\delta\rho_1(r') + \delta\rho_2(r') \right) dr'$, we can rewrite Eq. (29)

using the variations: $\delta\rho_1(r), \delta\rho_2(r)$:

$$\begin{aligned} & \int_0^\infty 4\pi r^2 dr \left[\left(\delta\rho_1(r) + \delta\rho_2(r) \right) dr - \left(\frac{\lambda_1 n_1(r) \delta\rho_1(r)}{\rho_1(r) + p_1(r)} + \frac{\lambda_2 n_2(r) \delta\rho_2(r)}{\rho_2(r) + p_2(r)} \right) B^{-1/2} \right] \\ & - G/c^2 \int_0^\infty 4\pi r^2 \left\{ \int_r^\infty 4\pi r' \left(\lambda_1 n_1(r') + \lambda_2 n_2(r') \right) B^{-3/2} dr' \right\} \left(\delta\rho_1(r) + \delta\rho_2(r) \right) dr = 0 \end{aligned} \quad (31)$$

where $B = \left[1 - \frac{2Gm(r)}{c^2 r} \right]$. In the last term, the integration order of r and r' was interchanged and the names were interchanged too. Since the variations $\delta\rho_1(r), \delta\rho_2(r)$ are arbitrary, equation (31) implies that in the region $r \leq R_1$, where the two species coexist

$$1 - \lambda_1 a_1(r) - \lambda_1 b_1(r) - \lambda_2 b_2(r) = 0, \quad 1 - \lambda_2 a_2(r) - \lambda_2 b_2(r) - \lambda_1 b_1(r) = 0 \quad (32)$$

where

$$a_i(r) = \left(\frac{n_i(r)}{\rho_i(r) + p_i(r)} \right) \left[1 - \frac{2Gm(r)}{c^2 r} \right]^{-1/2}, \quad i = 1, 2$$

$$b_i(r) = \frac{G}{c^2} \int_r^\infty 4\pi r' n_i(r') \left[1 - \frac{2Gm(r')}{c^2 r'} \right]^{-3/2} dr', \quad i = 1, 2$$

Equation (32) implies that

$$\lambda_1 a_1(r) = \lambda_2 a_2(r) \quad (33)$$

so that

$$1 - \lambda_1 \left(a_1(r) + b_1(r) + \frac{a_1(r)}{a_2(r)} b_2(r) \right) = 0$$

which in turn implies, using the facts that λ_1 and $\frac{a_1(r)}{a_2(r)}$ are constants

$$a_1(r)' + b_1(r)' + \frac{a_1(r)}{a_2(r)} b_2(r)' = 0$$

with a prime denoting an r -derivative.

Using the above equations results in

$$p_1(r)' = -\frac{G}{c^2} \left(\rho_1(r) + p_1(r) \right) \frac{m(r) + 4\pi r^3 \left(p_1(r) + p_2(r) \right)}{r^2 \left(1 - \frac{2Gm(r)}{c^2 r} \right)} \quad (34)$$

similarly we get

$$p_2(r)' = -\frac{G}{c^2} \left(\rho_2(r) + p_2(r) \right) \frac{m(r) + 4\pi r^3 \left(p_1(r) + p_2(r) \right)}{r^2 \left(1 - \frac{2Gm(r)}{c^2 r} \right)} \quad (35)$$

In the region $r > R_1$, ρ_1 and its variation are zero. Therefore, instead of Eq (32) one gets

$$1 - \lambda_2 a_2(r) - \lambda_2 b_2(r) = 0 \quad (36)$$

implying

$$a_2(r)' + b_2(r)' = 0 \quad (37)$$

which leads again to Eq (35), the equilibrium structure equation for species 2.

Equations (33) and (34) are the equilibrium structure equations that follow from the Einstein field equations. Thus, these equations are the necessary and sufficient conditions for the mass of the star to be stationary under arbitrary variations of the energy densities of the two species. In case that the second order variations are not zero, the stationary point is only an extremum: either a minimum implying a stable configuration or a maximum implying an unstable equilibrium.

In addition, equation (33) implies that in the region where the two species coexist

$$\frac{n_1(r)}{\rho_1(r) + p_1(r)} = \text{Constant} \frac{n_2(r)}{\rho_2(r) + p_2(r)} \quad (38)$$

which was obtained earlier (see Eq. (27)) as a result of the field equations and the covariant conservation laws.

V. AN ILLUSTRATIVE EXAMPLE

In discussion of neutron star equation of state(EOS), the role of nuclear forces is clearly important and is always an integral part of the discussion. We present here an example, where we employ the nuclear equation of state of Steiner, Lattimer and Brown [16]. It was obtained by fitting observational data of x-ray bursters to study the mixed neutron star. The dependence of the pressure and the number density on the energy density are displayed in Fig. 2. Thus, the maximal ordinary neutron star mass for this EOS is found to be $2.44M_\odot$ and the corresponding radius is 11.7 km .

Let us assume that $\Lambda' = \frac{1}{2}\Lambda$ where Λ' and Λ are the scales for the mirror and ordinary QCD, respectively. Since these scales largely control all masses, we expect that also the mass of the dark fermion, m_{b2} is half of the mass of the ordinary neutron, m_{b1} . For the dark baryons we use the same EOS scaled appropriately, so that the energy

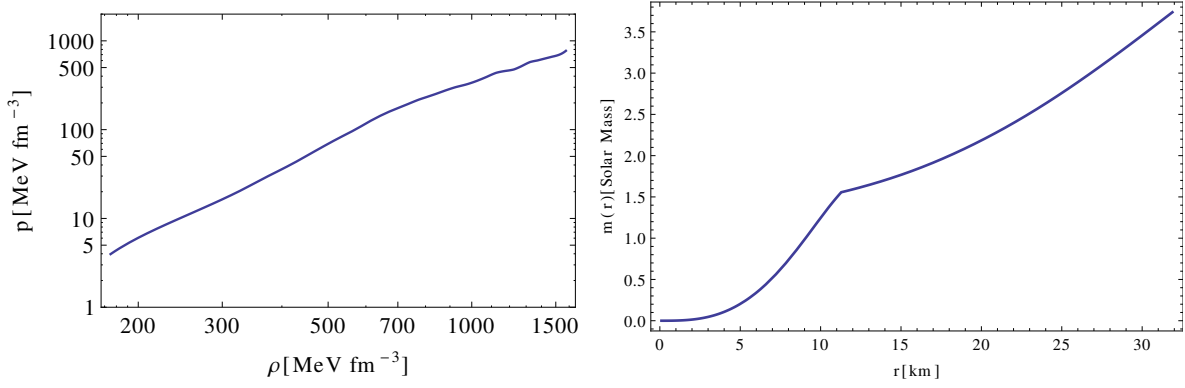


FIG. 2. Left: pressure as function of energy density. Units for both are $MeV fm^{-3}$. Right: Number density times mass in units of $g cm^{-3}$ as function of energy density in $MeV fm^{-3}$

densities and pressures scale as the fourth power of the corresponding masses. Thus, with this EOS, we have

$$p_2(\rho_2) = \frac{1}{16} p_1(16\rho_2) \quad (39)$$

The maximal mass of a pure dark neutron star is $\sim 10M_\odot$ and the corresponding radius is $\sim 50km$. It is expected that the mixed neutron star solution would yield a mass, and radius intermediate between those for a neutron star and a pure dark neutron star. We checked that the numerical results indeed obey equation (26). We also found that in accord with equation (26), indeed no static solutions are obtained when $\rho_1(0) > 16\rho_2(0)$.

We present an illustrative example of a typical mixed neutron star model for which: $\rho_1(0) = 600 MeV fm^{-3}$, and $\rho_2(0) = \frac{1300}{16} MeV fm^{-3}$.

The results of the computation are summarized in Table I. The r -dependence of the energy densities, the enclosed mass $m(r)$, and $\phi(r)$ are displayed in figures (3), (4).

VI. DISCUSSION

The section V shows that a mixed neutron star can have a total mass higher as measured by, say, orbital dynamics than pure, ordinary neutron stars. The radius, as probed by ordinary massless and massive particles, is the neutron-sphere radius which is similar in value to the radius of ordinary neutron stars as is the mass as measured by red

TABLE I. Model Results. The last entry is the gravitational binding energy of the neutrons inside R_1 , divided by the total neutron mass inside R_1 : $(N_1 m_b - M_1)/((N_1 m_b))$

M	M_2	M_1	$m(R_1)$
$3.74M_\odot$	$2.4M_\odot$	$1.34M_\odot$	$1.56M_\odot$
R_2	R_1	Redshifts	Neutron BE
31.9 km	11.1 km	$z(R_1)=0.72$, $z(R_2)=0.25$	22%

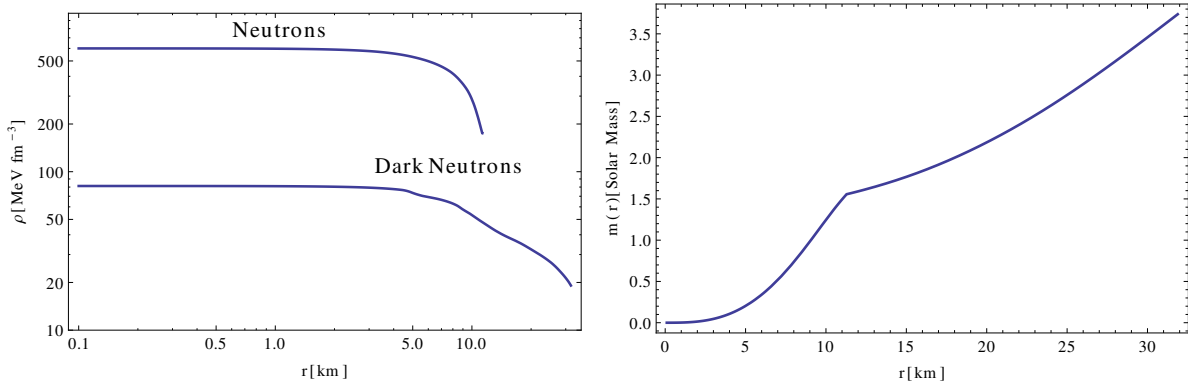


FIG. 3. Left: energy densities as function of radial distance. Right: Enclosed mass as function of the radial distance.

shift analysis.

There are implications for phenomenology of compact X-ray sources, related to the modified redshifts of emitted photons. This demonstrates that mixed neutron stars with masses exceeding those of ordinary neutron stars are possible and to study some of their general features. We now consider briefly the scenarios that can lead to the formation of such compact objects.

ADM can cluster without self-annihilation. Still in order for joint clustering to actually happen, further specific features are needed which may be difficult to build into complete consistent models. We will not discuss in detail how mixed stars may evolve but only sketch in broad terms how roughly equal masses of order of solar mass of baryons and

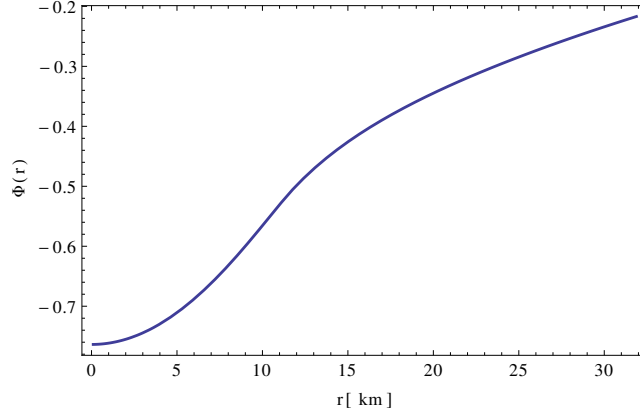


FIG. 4. $\phi(r)$ as function of radius. A photon emitted from the neutron surface R_1 , is redshifted by a factor 1.72 and a photon emitted from R_2 by 1.25.

dark fermions may possibly be brought together. Two general scenarios can be envisioned:

1. Dark matter is accreted onto ordinary stellar objects at various stages of the evolution of the latter, or conversely, ordinary matter is accreted onto pre-existing dark stars , and
2. Dark and ordinary matter jointly cluster forming the mixed stars

1. Accretion of DM onto stars has been discussed in the past [17–19]. It was motivated by noting that even tiny ($\eta_X = N_X/N_{Baryons} \sim 10^{-11}$) admixtures inside the sun of dark matter of mass $m_X \sim 5 - 10$ GeV can modify heat convection from the solar core and help explain some apparent anomalies. If the density of dark matter near the star has the average value of $0.4 \text{ GeV}/m_X \text{ cm}^{-3}$, then to generate $\eta_X \sim 10^{-11}$ over a Hubble time we need that σ_{XN} , the cross- section for scattering of dark and normal nucleons, exceed 10^{-37} cm^2 . This is excluded for heavy DM by direct searches - but not for the case of $m_X = 1/2$ GeV that we focus on in this paper. Furthermore once η_X exceeds the ratio of σ_{XN}/σ_{XX} (which can be as small as 10^{-15}), the nonlinear process of scattering the incoming DM on already captured X particles in the star dominates and further accelerates the accretion [18]. There is however an upper “unitarity” limit on the accretion rate fixed by the area of the star πR^2 (possibly with a “focusing” $(v_{escape}/v_{virial})^2$ enhancement ~ 10 for the sun) corresponding to the case of complete capture of all X particles which hit the

stellar surface. Even at this maximal rate if a solar type star were to accrete in Hubble time a solar mass of dark matter, we need that the DM density in its neighborhood will be 10^9 times larger than the local halo density of 0.4 GeV cm^{-3} . In general CDM starts clustering before baryons and our star may naturally be situated in a dark matter mini-halo. If this dark mini-halo formed at redshift z its density can be enhanced in comparison with the cosmological DM density of KeV/cm^3 by $(6z)^3 \sim 2 \times 10^8$ for $z \sim 100$. The CMB spectrum and simulations [20], certainly exclude forming mini-halos of solar mass at larger redshifts. Even then this achieves at most a 10^3 enhancement relative to the local halo density. It seems that only if DM was dissipative it could have clustered more effectively reaching the 10^9 enhancement required.

In passing, we note that the total DM accretion is not enhanced for bigger, more massive red/blue giant stars. The surface density M/R^2 of the such giant stars is smaller making it difficult to accrete the minimal amount of X particles required in order to initiate the non-linear regime and hence reach the unitarity limit. Furthermore the lifetime of these stars scales like M^{-3} making them live considerably shorter than solar systems. Also for more compact objects such as white dwarfs /neutron stars the enhanced focusing is offset by the far smaller areas.

Turning to collider constraints on dark matter (DM) properties in our model, the Atlas and CMS detectors at the LHC accelerator operate at unexplored energies and unprecedented rates. Their implications for DM properties stem from the fact that the detectors, triggered by large transverse momenta are ideal for detecting missing (transverse) energy. This underlies the remarkable, extensive SUSY searches at LHC as pair production of SUSY particles yields, often via spectacular decay chains, stable neutral lightest susy partner (LSP) s which escape the detector leaving an extra signature of missing transverse momentum. The failure to find any evidence for dark matter this way implies that dark matter of mass less than 100 GeV and O(weak) X-Nucleon scattering cross-section are excluded, if the production of $\bar{X}X$ pairs from a quark-anti-quark in proton-proton collision at LHC and the X-quark or the X-N scattering in direct underground DM searches proceed via a mediator heavier than both the ordinary and dark nucleon. The detailed analysis in [2] improves the rather weak bounds on WIMP nucleon elastic cross-sections obtained by direct underground searches for $m(X) \leq \text{GeV}$, the re-

gion of interest here, by up to six orders of magnitude below the Fermi constant . The above argument fails, and the bounds on the X-Nucleon cross-sections for the lighter DM candidates can be evaded, as we explain in the following if the mediator V of the X-q interaction is light. Specifically if V is lighter than the nucleon or the CDM particle X -or more generally than the invariant mass $M_{\bar{X}X}$ of the pairs in the above LHC signal events-we can no longer approximate the V exchange in the pair production process by a local four Fermi interaction . The X-Nuclear elastic scattering in the direct search experiments generate very small momentum transfers: $q = m_X \cdot \beta_X \sim 10^{-3} \cdot m_X = \text{MeV}$ for $m_X \sim \text{GeV}$. ($\beta_X \sim 10^{-3}$ is the typical virial velocity of the DM in the galactic halo). The scattering cross-section $\sim g^2 g'^2 [m_X^2 + q^2]^{-2} \sim g^2 g'^2 m_X^{-4}$ is therefore enhanced relative to the $\bar{X} - X$ production cross-section $\sim g'^2 g^2 m(\bar{X} - X)^{-4}$ with g, g' the coupling of the mediator V to $\bar{X} - X$ and $\bar{q} - q$) by factors of $\sim 10^4 - 10^8$ for $m_X = 1/2 \text{ GeV}$ and invariant mass $m_{\bar{X}-X} \sim 5 - 50 \text{ GeV}$. The non-observation at LHC of missing DM pairs cannot then constrain elastic cross-section in direct searches. A natural candidate for the light mediator V is the dark photon which kinetically mixes with our photon and is relatively light ($M_V \leq \text{GeV}$), and which has featured in [3] as well as in a class of CDM models in ref. [22] . To avoid as yet a far larger missing energy signal due to escaping dark photons the latter should decay in the detector into e^+e^- or $\mu^+\mu^-$ or pions and we should verify that those final states cannot be picked up at LHC and/or fixed target experiment as in Jeff lab [21]. Additional constraints that any DM scenario should satisfy, stems from the negative results of indirect searches for DM via γ rays from $\bar{X}X$ annihilations in over-dense regions and the galactic center in particular. The latter are readily satisfied here for two independent reasons. First for a predominantly asymmetric DM with only a tiny fraction of un-annihilated \bar{X} anti-particles remaining no further annihilations can happen. Second the total energy in $\bar{X} - X$ annihilations of only $\sim \text{GeV}$ implies that even the (rare!) two body annihilations $\bar{X} - X \rightarrow 2 \text{ photons}$ yield 0.5 GeV photons. The cosmic rays producing π^0 s in the atmosphere generate a very large γ background in which even the sharp 0.5 GeV line will be drowned.

With the LHC bounds inoperative, we can allow significant $\sim 10^{-30} \text{ cm}^2$ X-N cross-sections. A strict mirror model analogy suggests low energy XX cross-sections as high as $10^{-24} \text{ cm}^2 \sim \sigma(N, N)$ barely consistent with bounds suggested by observing some

elliptical halos which did not become spherical due to X-X collisions over cosmological times [23]. Gravitational clustering of CDM in most standard scenarios starts much earlier than for the baryons which then are still locked into the background radiation and only later after recombination fall into the potential wells generated by the DM particles. These CDM clustering have been studied in extensive n body simulations[20]. To our knowledge there are no similar calculations of joint clustering of ordinary and mirror baryons. It is possible that even the above cross-sections may be insufficient to jointly cluster ordinary and dark baryons. For some analogous studies of clustering of ordinary -and mirror matter, see [24] .

In general, dissipation is a key ingredient allowing significant clustering and shrinkage of matter/dark matter clouds. Mirror DM can be dissipative if the mirror photon mass $m_{\gamma'}$ is smaller than the atomic mirror excitations $O(m_{e'}\alpha^2)$ where e' is the mirror electron. As emphasized above, we should keep $m_{\gamma'} > 2m_e \simeq \text{MeV}$. Otherwise γ' becomes stable and can escape the collider leaving a missing energy signature as discussed above.

We can however have an alternative, more exact mirror symmetric dark matter scenario where the γ' is also massless and does not mix with normal photon where dark matter can be as dissipative as ordinary matter and joint clustering would then be possible. In such a scenario, we need to use a Higgs-type portal in order to generate $X - N$ interaction.

2. This leaves us with the second scenario where dark matter, while strongly interacting on its own, is non-dissipative . The idea is that the dissipative baryons that fall into the initial potential wells generated by the DM, will through gravitational interactions with the DM, dissipate also its energy. This then can allow dark and ordinary matter to jointly co-cluster into denser and denser structures so as to form eventually the mixed stars . This speculative scenario, however, should work only on relatively small scales, because for the galaxy as a whole dark matter is not clustered. Also favorable condition for this joint clustering may be relatively rare making mixed stellar objects of type being considered rather infrequent.

VII. SUMMARY

In summary, we have investigated the question of maximum neutron star mass if a substantial fraction of its mass is contributed by fermionic dark matter particles. We do this by solving the relativistic TOV equations using similar equations of state for ordinary baryons and dark fermions as well as with heuristic consideration of balancing kinetic thermal energy with gravitational energy of the two components (dark and ordinary baryons) of the neutron star. We find examples where, for the dark matter mass being half the neutron mass, leads to a neutron star mass two to four times higher than the Chandrasekhar mass. We also comment on possible scenarios where the required initial conditions for the abundances of the dark matter in the neutron star could arise. This work should be considered as a beginning attempt to get some ideas about the complex problem of two strongly interacting dark fermion species in a compact star and is meant to inspire future works on the subject. After a summary of our work appeared [10], this problem was also considered in [25] where it was argued that the presence of dark matter inside a neutron star softens the equation of state more strongly than hyperons, thereby changing its mass.

VIII. ACKNOWLEDGEMENT

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- [1] S. Nussinov Phys. Lett. B, 165, 55 (1985); S. M. Barr, R. S. Chivukula, E. Farhi, Phys. Lett. B241, 387-391 (1990); S. M. Barr, Phys. Rev. D44, 3062-3066 (1991); D. B. Kaplan, Phys. Rev. Lett. 68, 741-743 (1992); D. E. Kaplan, M. A. Luty and K. M. Zurek, Phys. Rev. D **79**, 115016 (2009) [arXiv:0901.4117 [hep-ph]]; for recent reviews and extensive list of references, see, H. Davoudiasl and R. N. Mohapatra, New J. Phys. **14**, 095011 (2012) [arXiv:1203.1247 [hep-ph]]; K. Petraki and R. R. Volkas, arXiv:1305.4939 [hep-ph].
- [2] P. J. Fox, R. Harnik, R. Primulando and C. -T. Yu, arXiv:1203.1662 [hep-ph].

- [3] H. An, S. -L. Chen, R. N. Mohapatra, Y. Zhang, JHEP **1003**, 124 (2010). [arXiv:0911.4463 [hep-ph]].
- [4] For a review and references to mirror models, see Z. Berezhiani, Int. J. Mod. Phys. **A**, **19**, 3775 (2004); L. B. Okun, Phys. Usp. **50**, 380 (2007) [hep-ph/0606202].
- [5] E. Kolb, D. Seckel and M. Turner, Nature **514**, 415 (1985); H. Hodges, Phys. Rev. **D 47**, 456 (1993); Z. Berezhiani, A. Dolgov and R.N. Mohapatra, Phys. Lett. **B375**, 26 (1996).
- [6] S. Tulin, H. -B. Yu and K. M. Zurek, arXiv:1302.3898 [hep-ph].
- [7] C. Kouvaris and P. Tinyakov, Phys.Rev.Lett., **107**, 091301 (2011), arXiv:1104.0382 [astro-ph.CO]; S. D. McDermott, H. -B. Yu and K. M. Zurek, arXiv:1103.5472 [hep-ph]; N. F. Bell, A. Melatos and K. Petraki, arXiv:1301.6811 [hep-ph]; C. Kouvaris and P. Tinyakov, arXiv:1212.4075 [astro-ph.HE]; see also I. Goldman and S. Nussinov, Phys. Rev. D **40**, 3221 (1989); A. de Lavallaz and M. Fairbairn, Phys. Rev. D **81**, 123521 (2010) [arXiv:1004.0629 [astro-ph.GA]]; C. Kouvaris, Phys. Rev. Lett. **108**, 191301 (2012) [arXiv:1111.4364 [astro-ph.CO]].
- [8] P. B. Demorest, T. Pennucci, S. M. Ransom, M. S. E. Roberts, & J. W. T Hessels Nature **467**, 1081 (2010); J. Antoniadis et al., Science, **340**, 448 (2013).
- [9] Lightman, A. P., W. H. Press, R. H. Price, and S. A. Teukolsky ; *Problem book in relativity and gravitation*, Princeton University Press, Princeton, NJ (1979); S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars: The physics of compact objects*, John Wiley Interscience (1983).
- [10] Some preliminary results of this investigation were presented by I. Goldman, APB, 42, 2203 (2011) [arXiv:1112.1505]
- [11] R. C. Tolman, *Relativity, Thermodynamics and Cosmology*, Oxford: Clarendon Press, (1934).
- [12] J. R. Oppenheimer and G. M. Volkoff Physical Review, **55**, 374 (1939).
- [13] I. Goldman Phys. Rev. **D40**, 327 (1989).
- [14] E.R. Harrison *Elementary Cosmology*, (Rutherford). Mar 1965. 50 pp. NIRL-LN-6.
- [15] S. Weinberg, *Gravitation and Cosmology*, John Wiley (1972).
- [16] A. W. Steiner, J. M. Lattimer, & E. F. Brown, ApJ, 722, 33 (2010).

- [17] D. N. Spergel, W. H. Press, *Astrophys. J.* **294**, 663-673 (1985).
- [18] M. T. Frandsen, S. Sarkar, *Phys. Rev. Lett.* **105**, 011301 (2010). [arXiv:1003.4505 [hep-ph]].
- [19] J. Faulkner and R. L. Gilliland, *Astrophys. J.* **299**, 994 (1985).
- [20] B. Moore, J. Diemand, J. Stadel, and T. Quinn (2005), astro-ph/0502213.
- [21] J. D. Bjorken, R. Essig, P. Schuster and N. Toro, *Phys. Rev. D* **80**, 075018 (2009) [arXiv:0906.0580 [hep-ph]]; S. A. Abel, M. D. Goodsell, J. Jaeckel, V. V. Khoze and A. Ringwald, *JHEP* **0807**, 124 (2008) [arXiv:0803.1449 [hep-ph]].
- [22] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer and N. Weiner, *Phys. Rev. D* **79**, 015014 (2009) [arXiv:0810.0713 [hep-ph]].
- [23] J. L. Feng, M. Kaplinghat and H. -B. Yu, *Phys. Rev. Lett.* **104**, 151301 (2010) [arXiv:0911.0422 [hep-ph]].
- [24] R. N. Mohapatra, S. Nussinov and V. L. Teplitz, *Phys. Rev. D* **66**, 063002 (2002) [hep-ph/0111381]; S. Nussinov, *Phys. Rev. D* **38**, 1606 (1988).
- [25] Ang Li, Feng Huang, Ren-Xin Xu, *Astroparticle Physics* **37**, 70 (2012).